AN EVALUATION OF THE TWO-CATCH METHOD OF POPULATION ESTIMATION

by

Burwell Gooch
Information Systems Division
Montana Department of Administration
Helena, Montana 59601

in cooperation with
Montana Department of Fish and Game
Helena, Montana 59601

October, 1977

An Evaluation of the
Two-Catch Method of
Population Estimation

October, 1977

Burwell Gooch

Introduction

The two-catch method is a simple technique for estimating the number of fish in a population (Seber, 1973). It is particularly handy for small populations living in relatively confined habitats. If the assumptions underlying the method are met, it can save considerable time and effort over more conventional techniques such as the mark-recapture method.

The method requires two runs through the population during which fish are captured (e.g., by electric shocking). These fish are held until the two runs have been completed, at which time they may be returned to the population. Population estimates are calculated as

$$\hat{N} = \frac{c_1^2}{c_1 - c_2} \qquad (1)$$

where

$\hat{N}$ = estimate of population N

c_i = number of fish caught during the ith run (i = 1,2)

Assumptions required by the method are:

1. The population is closed (no immigration, emigration, mortality, or recruitment) during the time between the two runs.
2. Catchability (the probability of a fish being caught) is the

same for both runs.

The first assumption is relatively easy to satisfy in the kind of situation for which the method is best suited (e.g., small streams). For example, block nets at upper and lower boundaries of the stream section will prevent immigration and emigration; conducting both runs on the same or succeeding days will minimize mortality and recruitment. The second assumption may be much more difficult to satisfy, depending on the mode of fishing. With electric shocking equipment, the probability of catch is known to vary as a function of size of fish, species of fish, and prior exposure to shocking. In the following, the relative effects of these factors are analyzed.

Size/Species Differences

In this section we will assess the influence of variation in catchability due to body size and/or species differences on the estimate of population size. To simplify the evaluation, the following assumptions are made:

1. The population consists of two groups of fish that may or may not contain the same number of fish.
2. Catchability of fish is constant within a group but may differ between groups.

The notation needed for the analysis is defined as follows:

$\hat{N}$ = estimate of total population N

$\hat{N}_i$ = estimate of i*th* group N_i (i = 1,2)

c_{ij} = catch of i*th* group of fish on j*th* run (j = 1,2)

P_i = catchability of i*th* group

$\alpha = N_2/N_1$

$\beta = P_2/P_1$

$\gamma = \hat{N}/N$

There are two possible choices for a population estimator $\hat{N}$: one based on the sum of individual group estimates, the other based on pooling the catch data of the two groups.

If

$$\hat{N} = \hat{N}_1 + \hat{N}_2$$

$$= \frac{c_{11}^2}{c_{11} - c_{12}} + \frac{c_{21}^2}{c_{21} - c_{22}} \qquad (2)$$

then it can be shown that $\hat{N} = N$ (ignoring sampling variation).

If

$$\hat{N} = \frac{(c_{11} + c_{21})^2}{(c_{11} + c_{21}) - (c_{12} + c_{22})} \qquad (3)$$

then it can be shown that $0 \leq \gamma \leq 1$ (ignoring sampling variation), where

$$\gamma = \frac{(1 + \alpha\beta)^2}{(1 + \alpha)(1 + \alpha\beta^2)}$$

Thus, estimates of total population size based on expression (3) are always less than or equal to the true value. Although expression (2) represents the better estimator from a theoretical standpoint, expression (3) is more likely to be used in practice for the following reasons:

1. It is easier to work with because it does not require any sorting of fish.
2. It is less subject to the vagaries of random sampling because it is based on larger numbers.
3. Moderate differences between the catchabilities of the two groups do not significantly bias the estimate (Table 1).

As illustration of the last point, we observe that if catchability of one group is no more than 50 percent greater than catchability of the other group ($0.67 \leq \beta \leq 1.50$), the worst estimate possible is only four percent low ($\gamma = 0.96$). Even if catchability of one group is up to twice that of the other ($0.50 \leq \beta \leq 2.00$), the maximum error in $\hat{N}$ is just eleven percent ($\gamma = 0.89$).

Change Due to Exposure

In this section we will examine the effect on population estimation of change in catchability from one run to the next. Although such change can result from one or more of several factors, we will assume here that the predominant factor is exposure to electric shocking on the initial run. Normally this results in a reduced catchability during the second run, with the amount of decrease being inversely related to the time between runs. For this analysis, the following assumptions are made:

1. All fish in the population are equally catchable during any one run.
2. Catchability decreases from the first run to the second run.

For this situation, the appropriate estimator of population size is given by expression (1). From this it can be shown that $P_1 \leq \gamma \leq 1$ (ignoring sampling variation), where

$$\gamma = \frac{P_1^2}{P_1 + P_1P_2 - P_2}$$

P_i = catchability on ith run ($i = 1,2$)

Thus, as in the previous section, we find that $\hat{N}$ will always be less than or equal to N. However, the effect of a change in catchability from one run to the next turns out to be rather more drastic than the effect of a difference between groups of fish. In fact, for common values of P_1 (≤ 0.5), decreases in P_2 result in comparable or worse underestimates of N (Table 2). For example, if $P_1 = 0.35$ and $P_2 = 0.25$, $\hat{N}/N = 0.65$. It is because of this circumstance that the two-catch method has very limited use in practical applications.

Another expression that leads to the same conclusion is

$$P_2 = \frac{2}{\gamma}(1 - \sqrt{1 - \gamma}) - 1 \tag{4}$$

This relation provides the minimum value of P_2 that guarantees attaining a predetermined value of γ. Expression (4) is tabulated in Table 3 for relevant values of γ. Here we see that in order to assure ourselves of a decent estimate of N ($\gamma \geq 0.9$, say), we must catch more than half of the population remaining at the time of the second run ($P_2 = 0.52$).

In those rare instances when catchability during the second run exceeds that during the first, three possibilities exist:

1. $1 < \gamma < \frac{P_1^2}{2P_1 - 1}$, for $P_1 > 0.5$ and $P_1 < P_2 \leq 1$

2. $1 < \gamma < \infty$, for $P_1 \leq 0.5$ and $P_1 < P_2 < \frac{P_1}{1 - P_1}$

3. $-\infty < \gamma < \frac{P_1^2}{2P_1 - 1}$, for $P_1 < 0.5$ and $\frac{P_1}{1 - P_1} < P_2 \leq 1$

In the last case the estimate of N is negative, so is obviously invalid. In the other two cases N is merely overestimated, so the bias may or may not be obvious.

Summary

The two-catch method of fish population estimation is particularly convenient to use in situations where the populations are small and time available for the two fishing runs is minimal. Although the method requires equal catchability of all fish in the population, deviations of up to 100 percent in a population consisting essentially of two groups do not lead to significantly biased estimates.

Unfortunately, when certain methods of fishing are used, such as electric shocking, catchability of the fish is moderately to greatly altered during the second run as a result of exposure during the first run. This in turn violates the requirement of equal catchability during both runs, and leads to severe underestimates of population size. As a result, the method cannot be recommended for general practice.

References

1. Seber, G.A.F. 1973. The estimation of animal abundance. Chas. Griffin & Co. Ltd., London. 506 pp.

Table 1. Values of γ [1/] as a function of α and β.

α \ β	0.10	0.20	0.50	0.67	1.00	1.50	2.0	5.0	10.0
0.1	0.93	0.94	0.98	0.99	1.00	0.98	0.94	0.58	0.33
0.2	0.87	0.89	0.96	0.98	1.00	0.97	0.91	0.56	0.36
0.5	0.73	0.79	0.93	0.97	1.00	0.96	0.89	0.60	0.47
1.0	0.60	0.69	0.90	0.96	1.00	0.96	0.90	0.69	0.60

1/ $$\gamma = \frac{(1 + \alpha\beta)^2}{(1 + \alpha)(1 + \alpha\beta^2)}$$

Table 2. Values of γ [1/] as a function P_1 and P_2.

P_1 \ P_2	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.60	0.70	0.80	0.90	1.00
0.10	1.00	-	-	-	-	-	-	-	-	-	-	-	-	-
0.15	0.35	1.00	-	-	-	-	-	-	-	-	-	-	-	-
0.20	0.33	0.50	1.00	-	-	-	-	-	-	-	-	-	-	-
0.25	0.36	0.45	0.63	1.00	2.50	-	-	-	-	-	-	-	-	-
0.30	0.39	0.46	0.56	0.72	1.00	1.64	4.50	-	-	-	-	-	-	-
0.35	0.43	0.49	0.56	0.65	0.79	1.00	1.36	2.13	4.90	-	-	-	-	-
0.40	0.47	0.52	0.57	0.64	0.73	0.84	1.00	1.23	1.60	4.00	-	-	-	-
0.45	0.51	0.55	0.60	0.65	0.71	0.79	0.88	1.00	1.16	1.69	3.12	20.25	-	-
0.50	0.56	0.59	0.63	0.67	0.71	0.77	0.83	0.91	1.00	1.25	1.67	2.50	5.00	-
0.60	0.64	0.67	0.69	0.72	0.75	0.78	0.82	0.86	0.90	1.00	1.13	1.29	1.50	1.80
0.70	0.73	0.75	0.77	0.78	0.80	0.82	0.84	0.87	0.89	0.94	1.00	1.07	1.14	1.23
0.80	0.82	0.83	0.84	0.85	0.86	0.88	0.89	0.90	0.91	0.94	0.97	1.00	1.03	1.07
0.90	0.91	0.92	0.92	0.93	0.93	0.94	0.94	0.95	0.95	0.96	0.98	0.99	1.00	1.01
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

[1/]

$$\gamma = \frac{P_1^2}{P_1 + P_1P_2 - P_2}$$

Table 3. Values of P_2 that guarantee a given value of γ or higher.[1/]

γ	P_2 [2/]
0.80	0.38
0.85	0.44
0.90	0.52
0.91	0.54
0.92	0.56
0.93	0.58
0.94	0.61
0.95	0.63
0.96	0.67
0.97	0.70
0.98	0.75
0.99	0.82
1.00	1.00

1/ For $0 \leq P_2 \leq P_1$

2/ $P_2 = \frac{2}{\gamma}(1 - \sqrt{1 - \gamma}) - 1$